THEORETICAL ANALYSIS OF CONDENSATION IN A ROTATING HEAT PIPE AT LOW ANGULAR VELOCITY OF ROTATION

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The mechanism of liquid layer formation is studied for condensation on the cylindrical wall of a rotating heat pipe in the low angular velocity range. Results of a numerical solution of the differential equation describing the condensation process are analyzed.

The majority of available studies of condensation on rotating heat pipes have been performed for high angular velocities of rotation, at which the condensing heat transport agent returns to the evaporation zone in the form of an annular layer along the longitudinal axis of the pipe. For this range of rotating heat pipe operation the differential equations describing the condensation problem have been obtained and solved [1], the condensate layer thickness distribution has been calculated [2], experimental dependences have been obtained for calculation of heat liberation coefficients [3], and methods for intensifying heat exchange have been studied [4]. The range of low angular velocities of rotation, where the condensate flows into the lower portion of the heat pipe and then moves into the evaporation zone in the form of streamlets is significantly less well studied and has been partially considered only in some experimental efforts [5-7]. The goal of the present study is a theoretical analysis of the process of liquid film formation and flow upon condensation in a horizontal cylindrical rotating heat pipe in the range of low angular rotation velocities.

In the velocity range under consideration coolant transport along the heat pipe axis occurs in streams due to a hydrostatic pressure differential along the length. Therefore outside the stream liquid displacement along the pipe axis may be neglected as compared to its motion along the wall in the transverse plane. Considering this, we will analyze the problem in a two-dimensional formulation. Using the assumptions of Nusselt theory, we write the motion and energy equations in the form

$$\nu\left[\frac{\partial^2 v_\varphi}{\partial \varepsilon^2}-\frac{1}{R-\varepsilon}\,\frac{\partial v_\varphi}{\partial \varepsilon}-\frac{v_\varphi}{(R-\varepsilon)^2}\right]=-g\sin\varphi, \tag{1}$$

$$\frac{\partial^2 T}{\partial (R-\varepsilon)^2}+\frac{1}{R-\varepsilon}\,\frac{\partial T}{\partial (R-\varepsilon)}=0. \tag{2}$$

The boundary conditions for Eqs. (1), (2) will be

$$v_\varphi=-\omega R,\ T=T_w \text{ for } \varepsilon=0,$$
$$\frac{\partial v_\varphi}{\partial \varepsilon}=0,\ T=T_s \text{ for } \varepsilon=\delta. \tag{3}$$

Integration of Eq. (1) with consideration of the assumption $\delta \ll R$ leads to the conventional parabolic velocity distribution over layer thickness:

$$v_\varphi=-\omega R+\frac{\rho g}{\mu}\left(\varepsilon\delta-\frac{\varepsilon^2}{2}\right)\sin\varphi. \tag{4}$$

The expression for the mean liquid velocity in the film will have the form:

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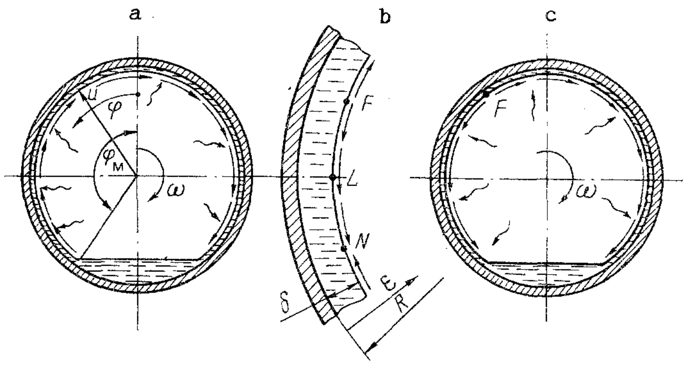

Fig. 1. Coordinate system and diagram of condensate motion in transverse section of rotating heat pipe: a) coolant condensed at any point outside stream moves in direction of tube rotation; b) moment of transition; c) boundary (point F) exists on liquid surface, from which liquid flows in opposite directions.

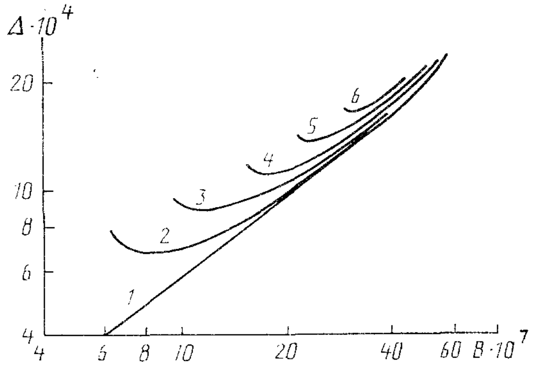

Fig. 2. Dimensionless liquid layer thickness at point $\varphi = \pi/2$ vs process parameters: 1) $C = 0$; 2) $0.5 \cdot 10^{-13}$; 3) $1 \cdot 10^{-13}$; 4) $2 \cdot 10^{-13}$; 5) $5 \cdot 10^{-13}$; 6) $1 \cdot 10^{-12}$.

$$\bar{v}_\varphi = \frac{1}{\delta} \int_0^\delta v_\varphi d\varepsilon = -\omega R + \frac{\rho g \delta^2}{3\mu} \sin \varphi. \tag{5}$$

Integrating the energy equation and substituting the expression obtained for the thermal flux density together with Eq. (5) in the equation for the increment in liquid flow rate per unit length of the surface element $d\varphi R$:

$$d(\rho \bar{v}_\varphi \delta) = \frac{q}{r} d\varphi R,$$

we obtain a differential equation describing the distribution of coolant layer thickness over the wall in the transverse plane of the rotating heat pipe:

$$-\omega R \rho \delta \delta' + \frac{\rho^2 g}{3\mu} \delta^4 \cos \varphi + \frac{\rho^2 g}{\mu} \delta^3 \delta' \sin \varphi = \frac{\lambda R \Delta T}{r}. \tag{6}$$

We use the notation:

$$B = \frac{\omega \nu}{gR}, \quad C = \frac{\lambda \mu \Delta T}{\rho^2 g r R^3}, \quad \Delta = \frac{\delta}{R}.$$

Then in Eq. (6) we can write in dimensionless form

$$-B\Delta\Delta' + \frac{1}{3} \Delta^4 \cos \varphi + \Delta^3 \Delta' \sin \varphi = C. \tag{7}$$

At low angular velocities of rotation, when a stress exists in the lower portion of the pipe, formation of a liquid layer on the heat exchange surface occurs due to entrainment of coolant from the stream by the wall, and because of the condensation process itself. If the value of ω is sufficiently large within the region under consideration, then the coolant condensed at any point of the surface outside the stream is captured by the entrained layer and removed into the stream in the direction of pipe rotation (Fig. 1a). In this case the boundary condition for Eq. (7) can be specified in the form

$$\Delta_0 = \Delta_e \text{ for } \varphi_0 = \varphi_m, \tag{8}$$

where Δ_y is the thickness of the entrained coolant layer at the point of exit from the stream, and φ_m is the angular coordinate of the liquid meniscus. With decrease in angular velocity the layer thickness on the ascending part of the pipe increases, while the velocity of the free liquid surface v_s decreases. This occurs up to the time when the value of v_s at the point $\varphi = \pi/2$ (point L of Fig. 1b) where the gravitational component $g \sin \varphi$ is maximal vanishes. Further decrease in ω causes the vector v_s at the point L to be directed downward. Above and below point L, in the positive and negative directions of the angle φ, the gravitational component decreases. For this reason in those zones there develop anew points (F and N of Fig. 1b) at which $v_s = 0$, between which there is a section where the liquid surface layer flows downward. Thus, from point F we have flow of the coolant both upward and downward, while it flows upward toward point N and enters the stream from below. As a result, an excess of liquid is formed at point N in the form of a cusplike thickening, which shifts downward with decrease in ω. With decrease in angular velocity the point F shifts in the negative φ direction (Fig. 1c) and in the limiting case ($\omega = 0$) occupies a position along the upper directrix of the pipe.

We will find the boundary condition for the case described, where there exists on the liquid surface a division point (point F), from which condensate moves in opposite directions. We write Eq. (7) in the following form:

$$\Delta' = \frac{C - \frac{1}{3}\Delta^4 \cos\varphi}{\Delta(\Delta^2 \sin\varphi - B)}. \tag{9}$$

For a nonmoving pipe ($B = 0$) we rewrite Eq. (9)

$$\Delta' = \frac{C - \frac{1}{3}\Delta^4 \cos\varphi}{\Delta^3 \sin\varphi}. \tag{10}$$

Hence it is evident that the film thickness in this case will have a finite value ($\varphi = 0$) under the condition

$$C = \frac{1}{3}\Delta^4 \cos\varphi. \tag{11}$$

Then the boundary condition for $\omega = 0$ will have the form

$$\Delta_0 = \sqrt[4]{3C} \text{ for } \varphi = 0. \tag{12}$$

Equation (12) is, in fact, the solution of the Nusselt problem of condensation on the outer surface of a nonmoving horizontal cylinder [8]. In this case the condensate flows to the left and right of the highest point F ($\varphi = 0$). It follows from Eq. (9) that for pipe rotation ($B \neq 0$) the indicated point corresponding to the coordinate at which the denominator of the right side of Eq. (9) vanishes shifts in the positive direction and occupies some position φ_0 ($0 < \varphi_0 < \pi/2$). The thickness of the liquid layer at this point will have a finite value upon satisfaction of the condition

$$C = \frac{1}{3}\Delta_0^4 \cos\varphi, \qquad B = \Delta_0^2 \sin\varphi. \tag{13}$$

Solving system (13) for Δ_0 and φ_0, we obtain the unknown boundary condition

$$\Delta_0 = \sqrt[4]{\frac{3C}{\cos\varphi}} \quad \text{for} \quad \varphi_0 = \arccos\left(-\frac{B^2}{6C} + \sqrt{\frac{B^4}{36C^2} + 1}\right). \tag{14}$$

As is evident from Eq. (14), for $B = 0$ the boundary condition thus found transforms to Eq. (12).

The mechanism of liquid layer formation for condensation on the inner surface of a rotating horizontal cylindrical pipe is an idealized model. In the real process the form and position of the boundary line passing through the indicated points will depend on various disturbing effects upon the liquid layer outside the stream. However, the proposed model permits us to analyze the effect of individual factors on the processes and clarify the principles of liquid layer formation and flow in the condensation zone of a rotating heat pipe.

We will consider the effect on the process of the parameters B and C, characterizing, respectively, the angular velocity of rotation and the intensity of condensation, using the example of numerical solution of Eq. (7) with boundary conditions (8) by a fourth-order Runge—Kutta method. Numerical values of boundary conditions (8) were defined by expressions obtained in [9]. Figure 2 shows results of the solution in the form of the dependence of dimensionless layer thickness Δ at the point $\varphi = \pi/2$ on the parameters B and C. The curves C = const have a minimum which corresponds to some rotation velocity exceeding which leads to increase in the total liquid film thickness due to increase in thickness of the entrained layer, while decrease below this value causes increase in Δ due to the dominating influence of the force of gravity as compared to friction forces. It is evident from the curves that the value of the temperature head ΔT defined by the parameter C affects formation of the liquid layer outside the stream differently for different B values. At high angular velocity within the limits of the range considered here (high B values) change in C has a weak effect on Δ. This is explained by the fact that the thickness of the film entrained from the stream is sufficiently large and its thermal resistance is significant. Moreover, at high angular velocity there is a rapid departure of the condensed coolant from the stream. For these reasons the fraction of the film formed by condensate in the total liquid layer outside the stream is negligible in this case and thus the effect on Δ, the change in this film thickness, due to variation of the parameter C is also small. At lower angular velocities (smaller B values) the entrained layer is thin and has a low thermal resistance while the rate of removal of the condensate formed is also low. As a consequence change in the parameter C is reflected more significantly in the quantity Δ, i.e., the process of condensation plays the main role in liquid layer formation outside the stream in this case.

Thus, in the process of condensation at low angular velocities of rotation, i.e., when a stream exists in the rotating heat pipe, we can distinguish the following characteristic regions of parameter relationships, distinguished by the principles of liquid film formation and motion: at any point outside the stream the condensed coolant moves in the direction of wall rotation; or there exists on the surface a division point, from which the coolant flows in opposite directions. In the first case formation of the liquid layer outside the stream is determined mainly by the process of entrainment of liquid by the rotating wall, with the layer thickness being described by Eq. (7) with boundary condition (8). In the second case the condensation process is dominant, and Eq. (7) is to be solved with boundary condition (14).

NOTATION

Here u and φ are polar coordinates; $\varepsilon = R - u$, auxiliary variable: ω, angular velocity of rotation; v, linear liquid velocity; R, radius of inner surface of rotating heat pipe; δ, liquid film thickness; g, acceleration of gravity; ρ, density; μ, ν, dynamic and kinematic viscosity coefficients; λ, liquid thermal conductivity; r, latent heat of vapor formation; T_w, wall temperature; T_s, vapor saturation temperature; $\Delta T = T_s - T_w$, temperature head.

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FEATURES OF THE BREAKUP OF JETS OF A LOW-VISCOSITY LIQUID IN A SUBSONIC ENTRAINING FLOW OF GAS

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The results of electrocontact measurements are used as a basis for examining the mechanisms (variants) of the breakup of a liquid jet and a dense atomizing jet. The deformational scheme of breakup in an entraining subsonic gas flow is generalized.

In the combustion of liquid fuel, a common practice is to inject a jet of the fuel at an angle into a subsonic flow of cold or heated oxidizing gas. The breakup of the jet is one of the primary acts of mixture formation and ultimately determines the dynamics of the combustion process [1, 2].

The breakup of drops has been examined in detail in the literature [1-10]. The atomization of jets by a supersonic entraining flow was described in [11-18]. Information on the breakup of jets injected at an angle to a subsonic flow was presented in [19-27]. These studies examined the mechanisms (variants) of jet breakup only as a means of explaining and approximating the results of measurements. At the same time, the subject of the mechanisms responsible for the disintegration of jets is of theoretical and, in particular, practical interest. For example, the gradual breakup of a jet [20-22] ensures the supply of fuel to the mixing zone. When the flame is stabilized on the surface of this zone, this situation leads to combination of the processes of mixing and combustion. The flame stabilization phenomenon itself is intimately connected with the breakup of the jet and is employed to provide for special regimes of operation of mixers [22, 26]. The catastrophic breakup of drops [2, 8, 9] or jets [13, 14, 23, 27, 28] contracts the mixing region and elevates its quality [27, 28]. Atomizing with the "stripping" of the surface layer of the liquid yields the finest droplets, which form a nearly homogeneous mixture with the gas [1, 2, 7, 8, 11]. By a variant of breakup, we mean the method by which the flow acts on the jet: "stripping" of a liquid film, excitation and destruction of waves, etc. The act of the separation of a drop from the body of a jet (ligament mechanism) was studied in [29, 30] and is not examined here.

The jet has an integral core which breaks up either gradually, as the jet enters the flow, or suddenly — in the case of catastrophic disintegration [20-23, 27, 28]. The mechanism of the jet's breakup depends on its path (trajectory) and the atomization surface (width of propagation) [20, 21, 25]. The inverse dependence of the degree of penetration of the jet on its breakup — substantiated theoretically in [12] — can be proven only by experiment. The electrocontact method is most informative in this regard, making it possible to determine the jet breakup function $f_b = R_j/R_0$ from the orifice to the point of disintegration of the core for any core trajectory (Fig. 1). The breakup function can be represented in the form

$$f_b = \frac{1}{l_e} \int \frac{F_0}{F} \, dl_e,$$

since

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